along the length of a sinusoidally shaped H-plane bend having $w/a = 7.05$ and $a/\lambda = 2.125$. The computer calculations indicate that for reasonably gradual bends the spurious mode voltages along the sinusoidally shaped bends increase monotonically from the bend input to the bend center, and then decrease monotonically from the bend center to the bend output. This variation should be compared with the cyclic variation obtained with the constant curvature bends.

Conclusions

The theoretical and experimental results indicate that reasonably compact constant curvature and variable curvature bends can be designed for rectangular waveguides having cross-section dimensions in the range between 1.5 and 2.5 free space wavelengths. Calculations indicate that low-mode conversion loss can be obtained over broad frequency bandwidths with the sinusoidally shaped bends.

The close agreement between the experimental and computer data indicates that accurate results can be expected with the calculated coupling coefficients and the coupled transmission line equations, provided all forward propagating modes are considered.

Acknowledgments

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Microwave Delay Techniques Using YIG

F. A. OLSON AND J. R. YAEGER

Abstract—Recent developments in microwave delay techniques employing single-crystal yttrium iron garnet (YIG) are described. In particular, the operation of a two-port, electronically variable-delay device utilizing long-wavelength spin-wave propagation in single-crystal YIG is presented in detail. Specific advantages of this device are transmission-type operation, delay continuously variable from zero to several microseconds by means of magnetic field, and lack of critical dimensions or surface finishes. This form of delay, as well as those due to acoustic-wave and spin-wave/acoustic-wave propagation, have been observed at frequencies from 1 to 10 Gc/s. A comparison of the performances of these delay processes is made, with special attention to insertion loss, bandwidth, frequency limits, and variable-delay range.

Introduction

The ability to delay microwave signals and, in particular, to control the delay electronically, is important to certain electronic systems. There has been considerable activity in this field recently, with the most significant noncryogenic results being achieved through the use of various delay processes in single-crystal yttrium iron garnet (YIG). A number of favorable conditions and phenomena suitable for microwave delay applications exist in a ferrimagnetic material such as YIG. The small magnetic loss in YIG is of particular interest here in that it allows reasonably efficient generation and propagation of magnetostatic waves and spin waves [1], [2]. This paper is mainly concerned with the application of long-wavelength spin waves (magnetostatic waves) to accomplish continuously variable delay [3]. The operation of this delay unit is compared with two other types of operation in YIG: fixed delay using low-loss acoustic waves [4], [5], and variable delay using combinations of acoustic and spin waves [6]. Special attention is given to delay range, insertion loss, bandwidth, and frequency limits of the delay processes.

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Variable Delay Utilizing Magnetostatic Waves

The propagation of magnetostatic waves in a longitudinally magnetized rod of YIG has been used to accomplish microwave variable delay. The magnetic waves result from the transverse microwave magnetic field applied at the end region of the YIG rod. The magnetic disturbance propagates through the crystal in the form of long-wavelength spin waves, and, because the medium is dispersive, the velocity of the waves can be controlled by varying the applied dc magnetic field in the dispersive region. Thus, variable delay is achieved by changing the group velocity of the waves rather than by varying the physical length of the propagation path.

A. Dispersion Theory

The general dispersion relation for spin waves in a ferrimagnetic ellipsoid magnetized along a principal axis is [7]

$$\omega_k^2 = \gamma^2(H_i + H_{ex}a^2k^2)(H_i + H_{ex}a^2k^2 + M \sin^2 \psi) \quad (1)$$

where

H_i = effective internal field
H_{ex} = exchange field
a = distance between neighboring spins
ψ = angle between H_i and k

For spin waves propagating in the direction of the magnetic field, this reduces to

$$\omega_k = \gamma(H_i + H_{ex}a^2k^2) \quad (2)$$

The usual dispersion diagram for z-directed spin waves is shown in the inset of Fig. 1. Since the group velocity of the waves is the slope of the dispersion curve, it is obvious that changing the operating point on the curve changes the velocity of group propagation. However, it has generally been unsuitable to operate in the region shown in this diagram because of the exceedingly short wavelength of the spin waves and the consequent difficulty associated with coupling.

In a longitudinally magnetized ferrimagnetic rod, the dispersion characteristic for long-wavelength spin waves is significantly different from that given by (2) which applies to short-wavelength spin waves. As described by Fletcher and Kittel [8], the dispersion relation for the complete spin-wave spectrum in a ferrimagnetic cylinder of infinite length is given by

$$\omega_k \approx \gamma \left\{ H_i + N_T M_S \left(\frac{x_i}{kR} \right)^2 + H_{ex}a^2k^2 \right\} \quad (3)$$

where R is the cylinder radius and x_i is a root of $J_0(x) = 0$. Figure 1 shows a graph of this dispersion relation, with a change in scale to emphasize the propagating magnetostatic waves. At large wave numbers the curve is the familiar spin-wave dispersion characteristic associated with exchange interaction as given by (2). At wave numbers less than about 3×10^3 cm^{-1} the curve represents propagating magnetostatic modes with backward wave characteristics, i.e., the phase and group velocities are of opposite direction. The group velocity comes from (3) and is given by

$$v_g = \frac{\partial \omega_k}{\partial k} = -2\gamma N_T M_S \left(\frac{x_i}{R} \right)^2 \frac{1}{k^3} \quad (4)$$

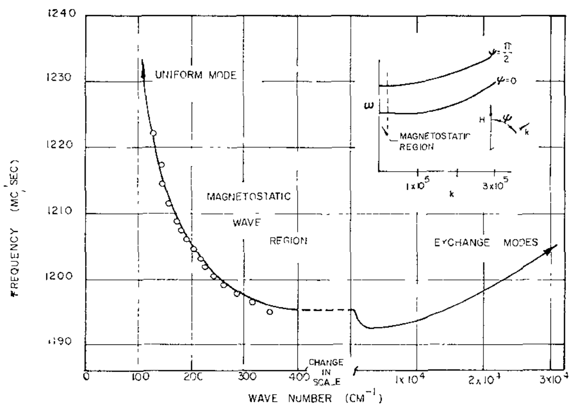

Fig. 1. Dispersion curve for z-directed magnetostatic and spin waves in YIG rod. *Inset:* Dispersion diagram for short-wavelength spin waves.

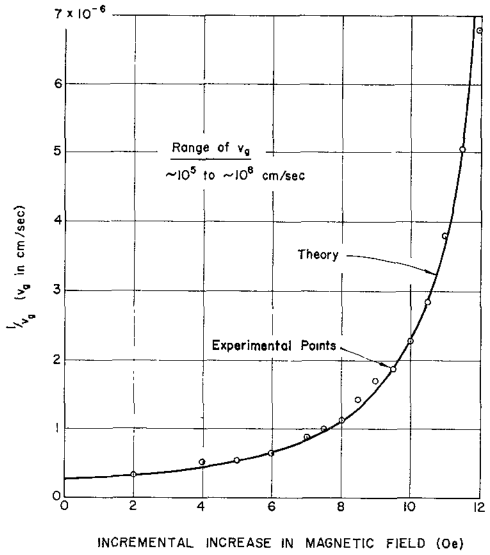

Fig. 2. Dependence of magnetostatic-wave group velocity on incremental magnetic field.

The variable-delay process described here utilizes these backward waves in the magnetostatic region of the dispersion characteristic. Changing the operating point by means of the magnetic field produces a change in the velocity of the waves. The dependence of velocity on incremental field is shown in Fig. 2 with included experimental points to be discussed later. The possible variation of velocity extends from approximately 10^8 cm/s down to a lower velocity extreme which is limited in practical situations to about 10^5 cm/s by the magnetic loss of the material. Using magnetostatic waves, an effective variation in delay of greater than two orders of magnitude can be achieved, and, because of the long-wavelength spin waves involved, relatively efficient coupling between electromagnetic and magnetic modes is possible.

B. Experimental Operation of Variable-Delay Line

The structure used in the magnetically variable-delay line is shown in the inset of Fig. 3. It consists basically of input and output couplers and a longitudinally magnet-

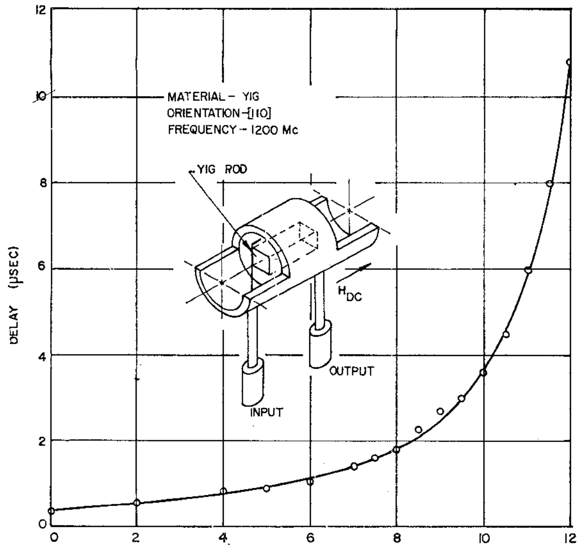

Fig. 3. Delay vs. control field. *Inset:* Structure used for investigating microwave delay processes in YIG rods.

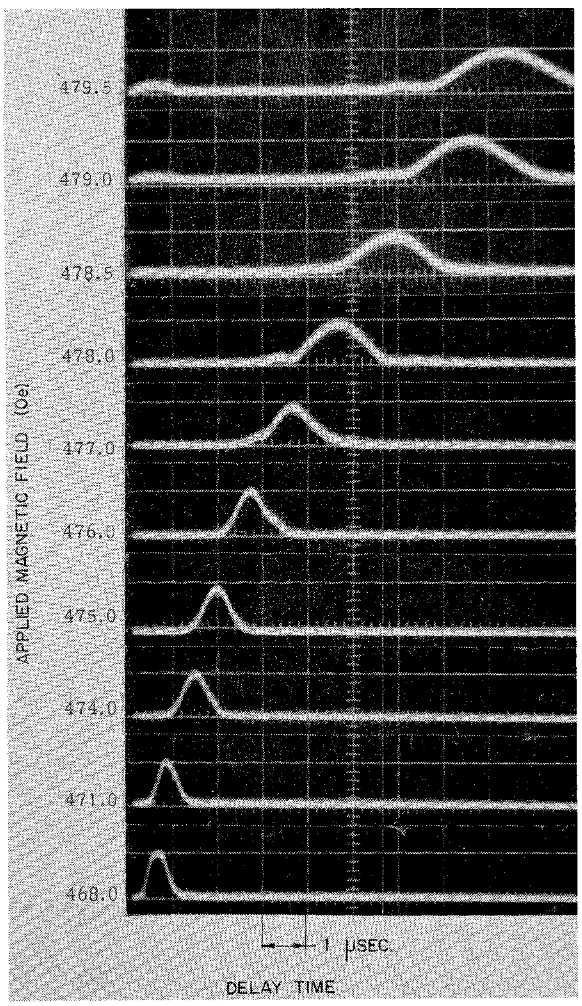

Fig. 4. Microwave delay, utilizing magnetostatic waves, at various magnetic fields. (Input level adjusted to maintain constant output pulse amplitude; frequency = 1200 Mc/s.)

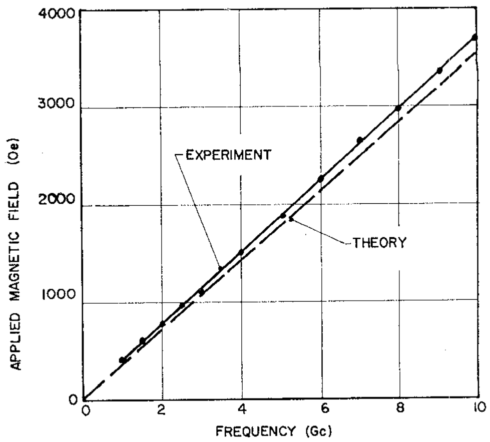

Fig. 5. Magnetic field required for 2-μs delay as a function of frequency.

ized YIG rod. An input signal is coupled to the end of the YIG rod by means of a thin-wire coaxial RF coupler. The magnetostatic wave propagates from the end region of the rod to the other end where output coupling occurs in a similar arrangement. By adjusting the biasing magnetic field, the group velocity and consequently the propagation time are varied.

Provisions were made for interchanging a number of YIG rods in the test structure. Single crystals of approximately 0.01 to 0.1 cm^2 cross-sectional area and 0.8 to 1.6 cm length were used. The samples tested included rectangular and circular cross sections, rod orientation along the [100], [110], and [111] crystalline axes, and polished and unpolished end faces. The experimental performance of the variable-delay line was not found to be critical on any of these conditions.

Fig. 3 presents a graph of the delay vs control field for a [110] oriented YIG rod at a frequency of 1200 Mc/s, and Fig. 4 shows typical oscilloscope traces of transmitted pulses through the magnetically variable-delay line. The signal delay can be varied from nearly zero to about 10 μs by increasing the magnetic field. Very small changes in the magnetic control field produce large changes in delay indicative of the high degree of dispersion in the propagation characteristic. Because the system is highly dispersive, the delay unit has a very narrow delay-bandwidth at a fixed value of magnetic field.

Associated with an increase in delay is an increase in propagation attenuation. The relative attenuation increases at a rate of between 3 and 10 dB/μs, depending upon the frequency range. The smaller slope value applies to the 1000-Mc/s region, the larger value to the upper microwave frequency range. The net broadband coupling loss at minimum values of delay is approximately 30 dB.

The extent of the tuning range of this variable-delay device and the wide bandwidth of the couplers are shown in Fig. 5. Here is presented the magnetic field required for 2-μs delay as a function of frequency, with a comparison of experimental results and theoretical predictions. The experimental results were obtained by utilizing an untuned coupling structure for the entire frequency range from 1 to 10 Gc/s. Thus, while the instantaneous delay-bandwidth of this delay device is relatively narrow, operation can be achieved over a very broad frequency band by using broadband couplers and tuning the dc magnetic field.

C. Discussion of Results

From measurements of transmission through the YIG rod, it is possible to determine the average group velocity and the approximate dispersion characteristics of magnetostatic waves. The order of the peaks of the magnetostatic mode spectrum, for swept magnetic field, confirms that the propagation mode is a backward wave, i.e., the phase and group velocities are of opposite direction. At the high-field end of the mode spectrum the magnetostatic waves form a transmission continuum. The large values of delay are observed near the edge of this continuum, where the waves are most dispersive.

The average group velocity of the magnetostatic waves has been calculated from the ratio of the crystal length to the measured delay time. The results as a function of control field are shown by the data points of Fig. 2. The agreement between the experimental and the theoretical values of velocity, when expressed in terms of incremental field (or incremental frequency), is good. Because the theory does not account for the nonuniform internal field in a finite-length sample, the degree of agreement must be considered fortuitous. Comparison in terms of absolute rather than incremental values of field is more difficult, owing to the nonuniform field within the sample which produces a large variation in the group velocity over the propagation path. More precise calculations that account for the effects of the field nonuniformity indicate that the major portion of the delay time occurs in the nearly uniform field region at the midpoint of the rod, even though this region extends over less than one-half the total length. These calculations, performed at only a few operating points because of the complexity involved, also show agreement between theory and experiment in terms of absolute values of magnetic field for rods with large aspect ratios.

Experimental comparison with the dispersion relation (3)—again in terms of incremental field or frequency, without including the effects of field nonuniformity—is shown by the data points in the dispersion diagram of Fig. 1. The data further substantiate that transmission through the YIG rod is by magnetostatic waves which propagate in a highly dispersive backward wave mode.

From insertion-loss measurements it is possible to estimate the relaxation time for the magnetostatic waves. We find $\tau \approx 10$ μs at 1200 Mc/s decreasing with increasing frequency. This value and the frequency variation are in approximate agreement with reported relaxation times for uniform mode and long-wavelength spin waves in single-crystal YIG [9].

Based on the theoretical and experimental results, there is good understanding of the underlying principles of operation of the variable-delay device employing magnetostatic waves. It is possible to predict the delay and insertion-loss variation with control field and with frequency. However, several details remain to be determined if the process is to be utilized effectively. In particular, more information is needed on the transverse mode pattern and on the influence of boundaries. This will allow the design of more efficient input and output couplers, and can result in the development of a microwave variable-delay device with considerably reduced insertion loss.

Comparison of Delay Processes in YIG

The variable-delay process utilizing magnetostatic-wave propagation is one of the three forms of delay which have been observed using YIG in the microwave frequency range:

1) Fixed-delay operation involving acoustic waves.
2) Variable-delay operation using combinations of acoustic and spin waves.
3) Variable-delay operation utilizing magnetostatic waves.

All three occur in the same general arrangement, but differ in detailed physical phenomena and delay properties. Figure 6 shows the longitudinally magnetized YIG rod common to all, and the variation in the internal magnetic field due to the demagnetizing factors of the rod-shaped sample. Also diagramed are the propagation paths of the particular waves that are involved in each delay process.

The first delay form shown in Fig. 6 utilizes acoustic-wave propagation, first reported by Spencer, et al. [4]. In this acoustic delay process an input signal produces a transverse microwave magnetic field which excites resonance at the rod face. Circularly polarized shear waves, generated at this face through magnetostriction, propagate the length of the crystal. When the waves arrive at the output end of the rod, part of the energy is coupled out by a similar process. Because coupling efficiency is relatively low, much of the wave energy is reflected at the output end and propagates back through the crystal. This multiple reflection produces output echoes which occur at fixed intervals determined by the physical length of the crystal. A typical example of recurring echoes from acoustic-wave propagation is shown in Fig. 7. It should be noted that acoustic-wave propagation in YIG originates at the crystal surface and requires precisely polished and aligned end faces.

Another delay process in YIG providing magnetically variable delay involves combined spin and acoustic waves, as first reported by Strauss [6]. This process, shown in Fig. 6, occurs at lower magnetic field values than for acoustic-wave propagation. The wave originates within the crystal where the nonuniform internal field of the rod produces a net magnetic dipole. A micro-

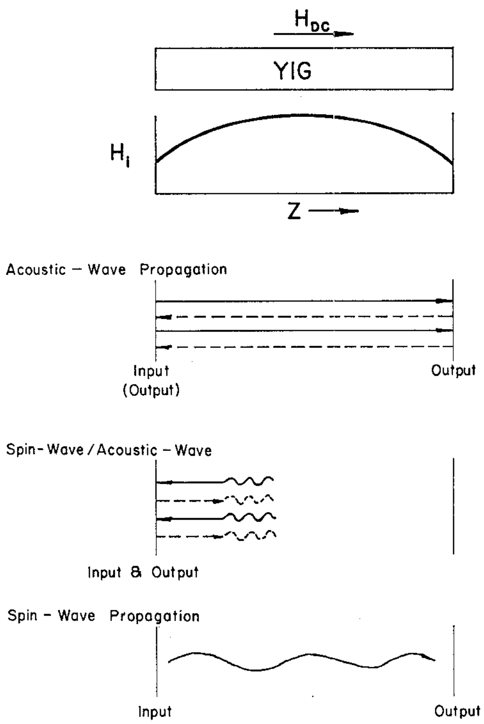

Fig. 6. YIG rod and internal field common to the three delay processes, with diagrams of the respective propagation paths.

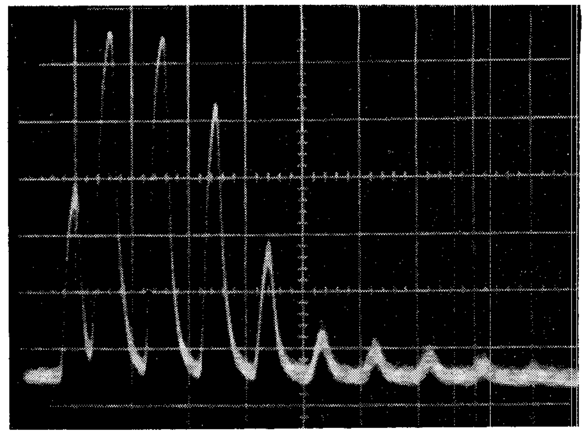

Fig. 7. Recurring echoes from acoustic-wave propagation in YIG rod. (Frequency = 1.9 Gc/s, field = 1310 Oe, Horizontal scale: 4 μs per major division.)

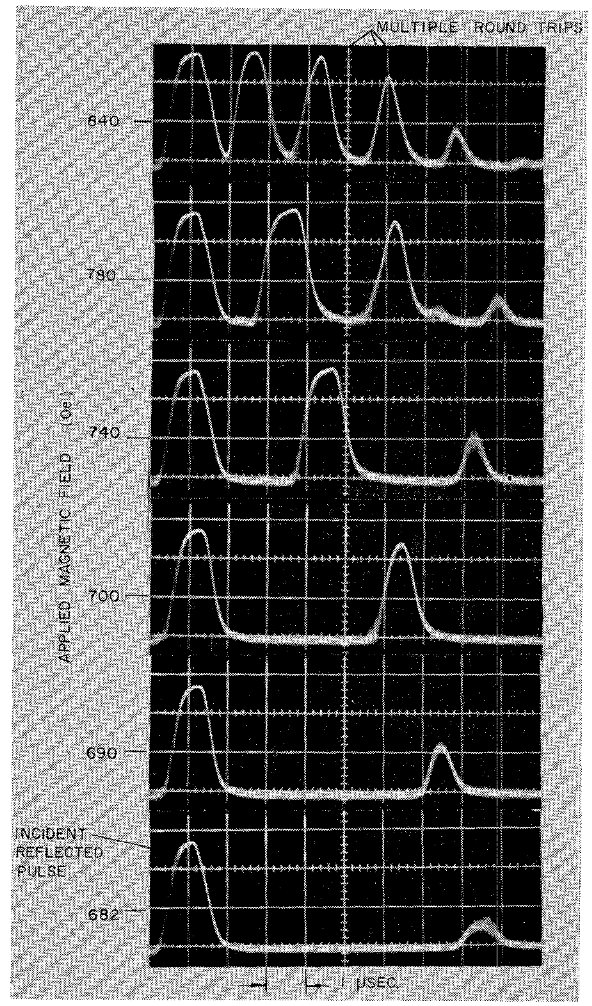

Fig. 8. Spin-wave/acoustic-wave delay at various magnetic fields. (Input level adjusted to maintain nearly constant output pulse amplitude; frequency = 1.6 Gc/s.)

wave magnetic field, coupled to this dipole by the usual thin-wire technique, produces a spin wave which propagates toward the nearest crystal end face. As the wave propagates toward the end, experiencing a decreasing magnetic field, it passes through the condition for magnetoelastic coupling where it is partially converted to an acoustic wave. The acoustic wave then propagates to the end surface of the sample. The acoustic wave is reflected at the surface, and returns through the magnetoelastic coupling region where it is converted to a spin wave; the spin wave then travels to the dipole coupling region where an output signal results. By changing the magnetic-field value, the point of coupling to the magnetic dipole can be changed, thus changing the length of the propagation path and also the proportion of propagation time devoted to either spin or acoustic waves.

The spin-wave/acoustic-wave delay process operates as a single-ended or reflection-type device with delay characteristics as presented in Fig. 8. The oscilloscope pictures show resultant delay from the spin/acoustic wave delay process at various magnetic fields. The bottom tracing is for the lowest magnetic field, the one at the top is for the highest magnetic field. From Fig. 8 it is apparent that delay decreases with increasing magnetic field. The large pulse at zero delay is the reflection from the coupling structure, a problem inherent in the use of reflection-type devices. At short values of delay, multiple round trips or spurious responses also become a part of this delay system. It is noteworthy that this process also requires polished end faces and has a minimum practical delay.

TABLE I
SUMMARY OF DELAY PERFORMANCE IN L-BAND USING YIG

Delay process	Insertion loss (dB)	Bandwidth (per cent)	Delay characteristics
Acoustic-wave	45	25	Recurring echoes at 2- to 8-μs intervals
	30 (cavity coupling)	0.1	Total delay ∼300 μs
			One- or two-port operation
Spin-wave/ acoustic-wave	37	*	Delay variable from 1 to 10 μs with magnetic field
	25 (cavity coupling)	0.1	Moderate-delay bandwidth, but tunable one-port operation
Spin-wave	30	*	Delay variable from 0 to 10 μs with magnetic field
	20 (cavity coupling)	0.1	Narrow-delay bandwidth, but tunable two-port operation

* Requires definition in terms of dispersion characteristics.

The third form of delay in YIG is the utilization of magnetostatic-wave propagation previously described in detail. The delay variation is accomplished by changing the group velocity of the waves along the propagation path illustrated in Fig. 6. The magnetostatic waves originate near the end region of the crystal (but not at the surface), propagate through the crystal, and are coupled out at the opposite end. There are no reflections since the crystal ends are poor magnetic reflectors; thus precisely polished and aligned ends are not required.

SUMMARY OF DELAY PROCESSES IN YIG

Table I summarizes insertion loss, bandwidth, and some of the delay properties of the three delay processes described in YIG. The insertion-loss values stated were obtained experimentally with the same coupling circuit for each delay process. This coupling structure is an untuned circuit capable of operation from 1 to 10 Gc/s. Consequently, the insertion-loss values obtained are realistic comparisons of the efficiency of the various delay processes. It is apparent also, in each delay process, that cavity coupling or tuned circuits can reduce the insertion loss but with accompanying decreases in usable bandwidth. The delay characteristics may be summarized as follows:

1) Acoustic delay is most suitable for long-term storage of pulses in a recurring-echo output fashion.
2) Spin/acoustic wave delay can provide moderate-delay bandwidth with variable delay from one to several microseconds, but with a minimum delay.
3) The spin-wave delay process yields delay variable from nearly zero to several microseconds, narrow-delay bandwidth, and freedom from reflections and spurious signals.

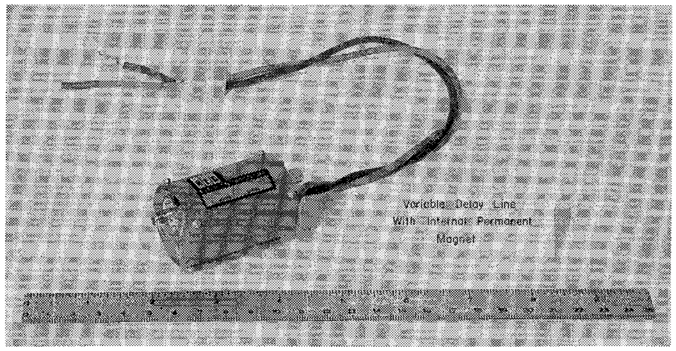

Fig. 9. Example of compact microwave delay device.

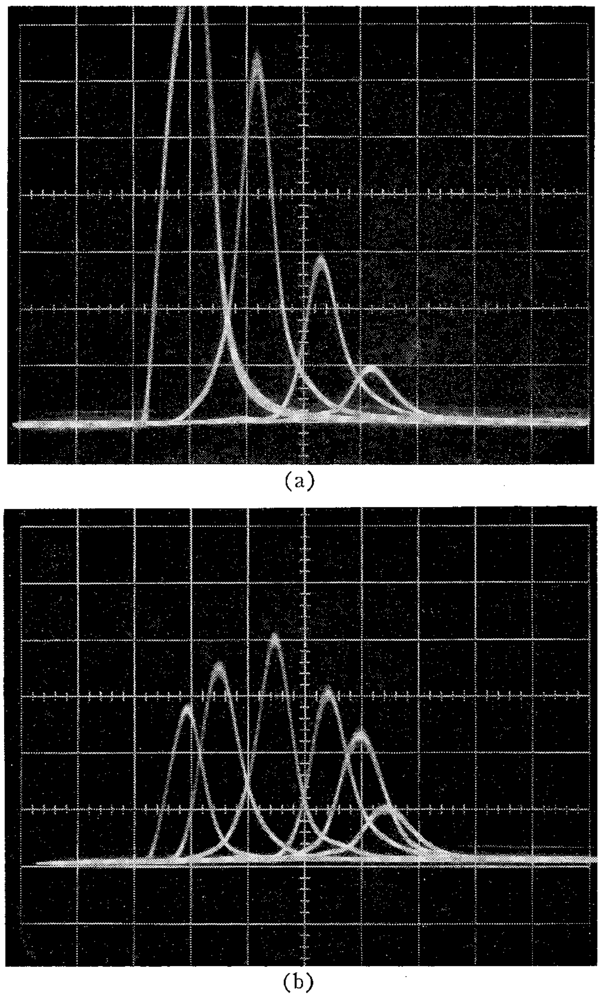

Fig. 10. Improved output of variable-delay unit when combined with compensating TWT amplifier. (a) *Top:* Multiple exposure showing variable delay of a microwave pulse. (b) Multiple exposure showing variable delay with compensating TWT amplifier. (Frequency = 1.5 Gc/s; field ≈ 560 Oe. Horizontal scale: 1 μs per major division.)

DELAY DEVICES

As an example of recent developments of microwave delay devices utilizing YIG, Fig. 9 shows a packaged delay-line unit suitable for any of the three forms of delay described. The unit is less than 2 inches long and 1.5 inches in diameter. It consists of a broadband coaxial input line and coupler, a YIG rod, and broadband coaxial output line and coupler. It has an internal permanent magnet and auxiliary tuning coil for producing variable delay, resulting in an extremely compact, integral delay unit.

One of the problems associated with either type of variable-delay operation is the inherent increase in attenuation with increasing delay. To relieve this problem, the delay unit was combined with a traveling-wave tube (TWT) amplifier to compensate for the insertion-loss change with increasing delay. A multiple-exposure photograph of the output of a delay unit utilizing magnetostatic waves is shown in Fig. 10(a). In this case the increasing insertion loss with delay is apparent. A multiple exposure of the output of the variable-delay unit when compensated by a TWT amplifier is shown in Fig. 10(b). By coupling the control coil of the delay unit and the gain electrode of the tube, variable delay was provided over 4 μs with a minimum change in output power. This kind of performance can be expected from many of the solid-state delay techniques by emphasizing the development of integrated delay units.

Conclusions

The operation of a two-port, electronically variable-delay device utilizing magnetostatic-wave propagation in a YIG rod has been demonstrated. The propagation, dispersion, and attenuation characteristics of the waves have been experimentally determined. The results substantiate previous theoretical predictions. The features of the variable-delay operation are transmission-type operation, delay continuously variable from nearly zero to several microseconds by means of magnetic field, and lack of critical dimensions or surface finishes. However, more detailed determinations of specific transverse modes and effects of boundaries should be made in order to optimize the design and performance of the device.

Delay processes involving acoustic waves and combinations of spin and acoustic waves have also been observed in YIG. The three processes were measured by use of the same coupling arrangement, thus allowing a valid performance comparison. Each delay technique appears useful in specific but separate applications. The integration of a delay device with a compensating amplifier has been shown to improve variable-delay performance substantially.

Acknowledgment

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